

XXXII. *On the Precession of the Equinoxes produced by the Sun's Attraction. By the Rev. Mr. Isaac Milner, M. A. and Fellow of Queen's College, Cambridge; communicated by the Rev. Dr. Shepherd, F. R. S.*

Read June 24, 1779.

IF the actions of the Sun and Moon upon the different parts of the earth were equal; or if the earth itself were perfectly spherical, and of an uniform density from the center to the surface; in either case the attractions of those remote bodies would have no effect on the position of the terrestrial equator, and the equinoctial points would constantly be the same in the heavens. But it was impossible to give the earth a rotatory motion round an axis without giving at the same time a centrifugal force to its parts. This force is greatest at the equator, and is in a contrary direction to that of gravity; on either side of the equator the force is less; and, moreover, only part of its effects is opposed to that of gravity. It is usual in determining the figure of the earth to consider the whole mass as in a state of fluidity, and the different columns as sustaining one another at the center. If the earth.

earth be considered as a hard body, firmly cohering in its parts by some other force besides that of gravity, it does not seem necessary that the different columns should be supposed to sustain each other at the center, though in both cases the direction of gravity must at every point of the surface be perpendicular to the tangent of the figure. But we know, that there is a considerable quantity of water upon the surface of the earth; and, therefore, if the equatorial regions were not higher than the polar, they certainly would be overflowed by the Ocean, which is contrary to experience; and for this reason the proportion of the diameters of the earth, determined upon the false supposition of an entire fluidity, cannot differ much from the truth.

§ 2. But the precession of the equinoxes, which depends upon the unequal actions of the Sun and Moon on the protuberant parts of the earth at the equator, will not be the same in these different hypotheses; at least we can never be certain that it will be so until we have computed their effects, and the computation itself must proceed on different principles. Suppose the earth to be fluid under the form of an oblate spheroid; or, what is more simple, suppose the region of the equator to be surrounded with a ring of fluid matter, and the unequal action of the Sun will disturb the figure of the ring, and
communicate

communicate a motion to its parts. Suppose we knew the precise disturbing force of the Sun upon any one particle of this ring according to its situation; in that case we could easily find the velocity which would be communicated to such a particle in any given time; but the mutual actions of the fluid particles upon each other could never be exactly estimated, much less their effects in endeavouring to turn the whole earth round its center. However, it is easy to see, that in the case of a hard ring of matter cohering close with the surface of the earth at the equator, both the law by which the particles act on each other, and on the whole mass of the earth, will be widely different from the case of fluidity, and the effects much greater in altering the position of the axis of rotation.

To explain this by an easy example (fig. 1,) let A, B, and c, represent three small bodies in the same horizontal line AE. Suppose A to descend by any accelerating force as gravity; B to descend by the same force, a less or a greater; and c not to be acted upon at all: in every one of these cases the bodies A and B will descend with their respective velocities, and the body c will preserve its situation. If A and B are small particles of fluid of any form, and c a hard one, and if the particle A be placed

in contact with B, have its center of gravity a little above the center of gravity of B, and is acted upon by the greater accelerating force; in this case we may conceive how the action of A may disturb the motion of B, and in the same way how the hard particle c may receive a small motion from the actions of A and B. Then this motion must be extremely little compared with the whole motions of A or B, and still a great deal less if c be strongly connected with a string of hard particles along the line CE, so that c cannot be moved without the whole line CE turning round the immoveable center E. Now if A, B, and c, be supposed hard particles firmly connected to the lever AE, then it is plain that the velocity of c, whatever it is, must be in proportion to that of A and B as their respective distances from E the center of motion, and this, whatever the impulsive forces are with which A and B are urged in their respective directions.

The body c being still supposed void of gravity, let the bodies A and B be urged by forces perpendicular to AE in any small equal times through the unequal spaces s and s' , and let the magnitudes of the bodies be represented by A, B, and c respectively. Then the space through which A is actually urged in that time will easily appear from mechanics to be represented by

$\frac{A \times AE \times s + B \times BE \times s \times AE}{A \times AE^2 + B \times BE^2 + C \times CE^2}$, and the space described by c is

$\frac{A \times AE \times s + B \times BE \times s \times CE}{A \times AE^2 + B \times BE^2 + C \times CE^2}$. See article the 13th.

§ 3. The preceding article being well understood, whatever doubts may remain concerning the motion of a ring of matter considered as detached from the earth, we may be certain that the motion of the nodes of the equator can never be the same, whether we suppose the ring at the equator to be fluid and to rest upon the surface of the earth, partaking of the diurnal motion, or whether we suppose it hard and compact, and by its cohesion communicating a proportional degree of motion to the different parts of the earth. In fact, the problem of the precession of the equinoxes, which has hitherto been considered as extremely difficult, and in its solution drawn out by authors to an immeasurable length, requires no principles but the received doctrine of motion, and the application of the lever, which have been made use of in the last article. In that article we supposed the bodies A and B to be impelled by different forces in parallel lines, and we estimated the real space, which either A or c in any small time would describe in consequence of those impulsive forces and their mutual connection by an inflexible lever. Now this is precisely what is re-

quired to be done in the case of the Sun's unequal action on the protuberant parts of the equator. The excesses or defects of that unequal action are to be considered as forces applied to those parts, which would move them according to the different circumstances through unequal spaces proportional to the forces in equal times of action, provided the particles were at liberty to move freely in the directions in which they are urged; and, lastly, the real space must be computed through which a particle moves at some known distance from the center of the earth in consequence of these various forces. This whole process will not differ from the easy example already described, except in the length of the calculation, and the proper management of the doctrine of fluxions; and it seems advisable in difficult subjects always to begin with simple instances before we proceed to those which are more complex, and to distinguish the algebraical operations from the principles upon which they are founded.

§ 4. In order to determine how much any particle of the earth is affected by the unequal action of the Sun (fig. 2.), let $CADB$ represent the earth, s the Sun at a great distance, and CD a plane perpendicular to the line ST joining the centers of the Sun and earth. If SK or ST represent the accelerating force of the Sun on a particle at the earth's center, and SL be taken to SK in the duplicate ratio

ratio of ST to SP ; SL will represent the attraction on any particle P , and by the resolution of motion Tm or PL will represent the perturbing force of the Sun on the same particle. By the construction $SL : SK :: SK^2 : SP^2$, and by division $KL : SK :: SK^2 - SP^2 : SP^2 :: SK + SP \times PK : SP^2$ and PL or Tm is nearly equal to $3PK$, and as $3PK$ is to SK or ST , so is the space described by P in any small time in the direction PK , to the space described in the same time by the center of the earth in consequence of the Sun's attraction. This last space is equal to $\frac{x^2}{2ST}$ where x represents the arc described by the earth's center during any small motion in its orbit, and the former is equal to $\frac{3PK \times x^2}{2ST^2}$. This is the space which would be described by P in the direction PK if the particle was at liberty to move freely. Let us at present suppose that no other particle is disturbed by the Sun's attraction except this one, and then proceed to enquire into the effects of this disturbance when P by its cohesion communicates a motion to the different parts of the earth, which is farther constrained to turn round an axis T , the common intersection of the plane CD and the terrestrial equator. From the laws by which motion is communicated, and the property of the lever, it easily appears, as in the second article, that the space through which any particle of the

earth's equator is impelled at the greatest distance from the axis τ , is to $\frac{3PKz^2}{2sT^2}$, the space which would be described in the same time by any particle at liberty, the magnitude of which is represented by P , as $P \times KT \times$ the radius of the equator to the sum of all the particles of the earth multiplied into the squares of their respective distances from the said axis.

To compute this sum in the easiest way, and by an approximation, which is quite sufficient when the polar and equatorial diameters differ little from one another; let DPE (fig. 3.) be a sphere whose radius is unity, divided into an infinite number of thin cylindrical surfaces, whose bases are the circles NAQ ; it is obvious, that all the particles in any one of these surfaces are at the same distance $CA = x$ from the axis of motion perpendicular to the plane of the circle NAQ . Call AP, y , and A , the area of the circle DPE and the fluent of $4Ax^3\dot{x}y$, or of $-4Ax^2y^2\dot{y}$, because $x\dot{x} = -y\dot{y}$ gives the sum of all the particles in the sphere multiplied into the squares of the respective distances from the axis. This fluent corrected is equal to $\frac{8A}{15}$, and must now be diminished in the ratio of 1 to $1 - 2p$, if we suppose the earth to be an oblate spheroid whose equatorial diameter is to the polar as 1 to $1 - p$;

4

and,

and, lastly, the space described by a particle at the greatest distance from the axis is equal to $\frac{45P \times PK \times KT \times z^2}{16A \times ST^2 \times 1 - 2p}$.

§ 5. In fig. 4. let $PIA\hat{p}DK$ represent the earth orthographically projected on the plane of the solstitial colure; P, p , the poles, IK a lesser circle parallel to the equator, and $pape$ a sphere described with the polar radius PT : then, since the particles without the globe only are concerned in changing the position of the axis of rotation, let L represent such a particle situated in the circumference of the circle IK , and by the preceding article its effect will be $\frac{45L \times LM \times MT \times z^2}{16A \times ST^2 \times 1 - 2p}$, and by the same way of reasoning, when two equal particles L, l are supposed to be disturbed by the Sun's attraction, the space described by that point of the equator, which is at the greatest distance from the axis of rotation or the common intersection of the plane CD and the equator A , will be equal to $\frac{45z \times L \times LM \times MT + l \times lm \times mt}{16ST^2 \times A \times 1 - 2p}$, and the same argument holds for every other particle without the sphere.

The sum of all the $L \times LM \times MT + \&c.$ must now be found; and for this purpose Sir ISAAC NEWTON'S construction is, perhaps, as convenient as any that has hitherto appeared. In the same figure Nn is parallel, and xy perpendicular, to CD ; take $Lx = x l$, and let m, n , represent
sent

sent the sine and cosine of the angle CTP to the radius unity. It is easy to prove in his way that $L \times LM \times MT + l \times lm \times mT$ is equal to $2L \times m \times n \times \overline{Lx^2 - Tx^2}$, and the fluent of Lx^2 multiplied into the fluxion of the circular arc Lx is easily found in the following manner, without having recourse to tables of fluents, or the methods of continuation.

From a known analogy the fluxion of the arc Lx is to the fluxion of its verfed sine as the radius Lx of the same circle to Lx the right sine. Lx multiplied into the fluxion of the verfed sine is the fluxion of the area of the semi-circle LL , and calling Lx, y , the fluent of Lx^2 multiplied into the fluxion of the arc Lx is evidently equal to $\frac{y^3 A}{2}$, where A still represents the area of a circle whose radius is unity: Ay is equal to the semi-circumference IK and $Ay \times Tx^2$ is equal to the fluent of Tx^2 multiplied into the same fluxion, and calling Tx, v , and substituting for Ii its equal py , the sum of all the $L \times LM \times MT + \&c.$ in the annulus Ii is equal to $mnpA \times y^4 - 2y^3 v^2$. This last quantity multiplied into the fluxion of v , and the fluent taken by the common method when v is equal to TP or unity nearly, comes out $\frac{4pAmx}{15}$ and twice this quantity gives the sum of all the $L \times LM \times MT$, without the whole sphere pap , and therefore the space described by a particle of the equator

tor in the circle of the Sun's declination while the center of the earth is carried through the space \dot{z} of its orbit is equal to $\frac{3pmn\dot{z}^2}{2sT^2 \times 1-2p}$, and may be supposed to be equal to $\frac{3pmn\dot{z}^2}{2sT^2}$, the alteration produced by the correction in art.

4. on account of the spheroidal figure of the earth being too inconsiderable to affect the conclusion.

§ 6. We are to observe, that the space $\frac{3pmn\dot{z}^2}{2sT^2}$ described by that point of the equator, which is the intersection of the circle of the Sun's declination, is generated by the perpetual attraction of the Sun. This attraction may be reckoned constant during the very small time of the earth's describing \dot{z} in its annual motion; and, therefore, the said point of the equator, at the end of that time, will have acquired a velocity which would carry it through $\frac{3pmn\dot{z}^2}{sT^2}$ in the same time.

§ 7. Let τ represent the time of the earth's revolution in its orbit, t the time of its rotation round its axis, and suppose \dot{w} to be a small arc similar to \dot{z} in a circle whose radius is unity. In figure 5. let AQ be the equator, and take Ab equal to $\frac{\dot{w}\tau}{t}$, and $b\dot{t}$ perpendicular to Ab equal to $3pmn\dot{w}^2$, and $Ab, b\dot{t}$, will represent the directions and quantities of the two different motions of the point A , and consequently $A\dot{t}$ will be the direction of the

new equator, and as Ab or At is to bt , so is the radius unity to the sine of the angle tAb , and if AQ or AG be taken equal to a quadrant, GQ the measure of the angle GAQ is equal to $\frac{3pmnw}{T}$.

§ 8. Suppose s the Sun's place in the ecliptic NS , N the equinoctial point, NA the Sun's right ascension, and ro a perpendicular on AN : then ro is to GQ as the sine of AN to the radius, and rN to ro as the radius to the sine of the angle at N the inclination of the ecliptic to the equator, and, *ex æquo perturbatè*, rN to GQ as the sine of AN to the sine of N and rN the small precession of the equinoxes is equal to $3pmn \frac{w \times t}{T} \times \frac{\sin. NA}{\sin. N}$.

§ 9. In the spherical triangle ASN , the sine of AN = $\cotang. N \times \tang. AS = \frac{m \times \cos. N}{n \times \sin. N}$, and farther the sine of $SN = \frac{m}{\sin. N}$ and rN is equal to $\frac{3pt \times \overline{\sin. w}^2 \times w \times \cos. N}{T}$, whose fluent or $\frac{3pt}{4T} \times \overline{2w - \sin. 2w} \times \cos. N$ gives the precession of the equinoxes during the Sun's motion through the arc NS of the ecliptic: when NS is equal to a circle, then the whole fluent becomes equal to $\frac{3ptA}{T} \times \cos. N$, and as wT is to $3pt \times \cos. N$, so is $60 \times 60 \times 360$ to $21'' + 6'''$ the annual precession of the equinoxes in seconds produced by the Sun's attraction.

§ 10. We might now proceed in a similar way to investigate the effects of the Moon's disturbing force, the rotation of the earth's axis, and the equation of the precession; but since these propositions are purely mathematical, and the computations have already been gone through by other authors, it will be needless to repeat them here.

§ 11. NEWTON was the first who attempted to explain the precession of the equinoxes from its causes. Since his time various other solutions have been given us by the most celebrated mathematicians; and it deserves to be noticed, that, in a case where there can be little doubt that he was mistaken, other authors have found it difficult to agree among themselves in differing from him. M. D'ALEMBERT, in the year 1749, printed a treatise expressly on the subject, and has since said ^(a), that himself is acknowledged to be the first who determined rightly the method of solving such problems. EULER, DE LA GRANGE, FRISIUS, SILVABELLE, WALMESLEY, SIMPSON, EMERSON, have each considered the subject, and perhaps the importance of the enquiry would justify a minute examination into the cause of the agreement or disagree-

(a) D'ailleurs, des géometres, vraiment capables d'apprécier mon travail, ont abondamment suppléé à tout autre témoignage, en déclarant que j'ai ouvert le premier la route pour résoudre ce genre de questions. See Opusc. Math. vol. V. sur la Précession des Equinoxes.

ment of their several methods; but I am deterred from entering into such a discussion by the length of time which it would require; especially as I think those who have read the authors mentioned will easily conceive the substance of what I should have to observe, and to those who have not read them I should hardly be able to say any thing intelligible.

§ 12. The above solution, if it had no other advantages, is, I apprehend, much more concise than any that has hitherto been given. Abstracted from what is said by way of illustration, articles 4th to 9th contain all the calculation requisite, and as I have studiously avoided the ambiguous use of the terms *force*, *vis*, *efficacia*, *momentum*, &c. as well as every doubtful representation of times, spaces, and velocities, which are often substituted by authors in equations, I believe the whole process will appear easy, and the evidence upon which the conclusion rests be exactly ascertained.

§ 13. The principles described in articles 2. and 4. depend upon the third law of motion, and the property of the lever, and are demonstrated in the following manner. Every thing remaining the same as in art. 2. (fig. 6.) let AV and BR, perpendicular to the right line or axis AE, represent the forces and directions with which those bodies are respectively urged, when at liberty to move

freely in those directions; and let Av , Br , Cc , represent the accelerative forces of the respective bodies, as altered by their mutual actions upon each other: then, because $C \times Cc$ is the moving force gained by C , and $A \times vV + B \times rR$ the moving force lost by A and B , regard being had to the lengths of the different levers AE , BE , we shall have $A \times vV \times AE + B \times rR \times BE$ equal to $C \times Cc \times CE$, that is, $A \times AE \times Av - Av \times B \times BE \times Br - Br$ equal to $C \times Cc \times CE$, and by transposition $A \times AE \times Av + B \times BE \times Br$ equal to $C \times CE \times Cc + A \times AE \times Ar + B \times BE \times Br$. Let s , s' , represent, as in art. 2. the spaces which would be described by the bodies A and B at liberty in any very small portion of time, and let x be the space which A actually describes in that time when connected with B and C by the lever AE . The quantities $\frac{x \times BE}{AE}$, $\frac{x \times CE}{AE}$, will then be the spaces described by B and C respectively; and, lastly, because the spaces described in given times are as the accelerating forces, the

above equation gives x equal to $\frac{A \times AE \times s + B \times BE \times s' \times AE}{A \times AE^2 + B \times BE^2 + C \times CE^2}$.

The same method extends itself easily to more difficult cases, and by its assistance several very important theorems are briefly demonstrated.

§ 14. The reasoning made use of in art. 6. will appear very evident to any one moderately versed in the

elements of mechanics and the doctrine of moving forces; and therefore I must believe that it is by mistake that one author of note entirely omits so necessary a step which affects the conclusion by just one half. When a body moves with any velocity in the direction AM (fig. 7.) which would carry it through the space AD in a small particle of time, and any force which may be reckoned constant for that time urges the body through the space DC perpendicular to AM , the body at the end of that time will arrive at the point c ; but joining AC we are not to suppose, that, if that force ceased to act, the body would proceed in the direction ACL : for take cm equal and parallel to AD , and cd in CD produced equal to $2CD$, and the direction of c at that point will be cl , the diagonal of the parallelogram $cdlm$.

Thus when a body revolves in any curve by a centripetal force (fig. 8.) we may, with Sir ISAAC NEWTON, suppose the curve to be composed of an indefinite number of right lines, and the body to move either in the chords or the tangents of the curve; but then we are to take care that we make not suppositions inconsistent with each other. Let the curve be a circle, and AD a tangent at the point A the direction of the body's motion when it arrives at that point, and let DC , parallel to the diameter AL be the effect of the centripetal force: then, if we

suppose the body to move along the chord AC , and say, that the angle CAD measures the deflection of the path in the time of the body's moving through the arc or chord AC , we shall mistake by one half of the true quantity; for draw the tangent at c , and since Ad is equal to dc from the property of the circle, the angle cDd of deviation is equal to twice the angle $CA d$. The practice of NEWTON in a similar case, where he is investigating the horary motion of the lunar nodes in a circular orbit, is entirely consistent with this. See the Principia, lib. III. prop. 30.

§ 15. M. D'ALEMBERT has lately charged SIMPSON's account of the precession of the equinoxes with some mistakes of this nature in his second lemma; but, in justice to SIMPSON, I must say, that, whatever other defects there may be in his paper, I am convinced, after the most diligent attention, that those alluded to are without foundation.

§ 16. SIR ISAAC NEWTON first observed, that an homogeneous globe could not possibly retain many distinct motions, without compounding them all into one, and revolving with a simple and uniform motion about an invariable axis. When two forces impress upon a globe two distinct circular motions ^(b), he briefly concludes in

(b) See Principia, lib. I. prop. LXVI. coroll. 22.

his way, from the laws of motion, that it is the same thing as if those two forces were at once impressed in the common intersection of the equators of those motions, and upon this principle we supposed At in art. 7. to be the direction of the new equator. In order to remove any doubts that might arise about the justness of this mode of compounding motion, FRISIUS has given a geometrical demonstration of the principle: but the thing may be shewn much more easily in the following manner. Suppose (fig. 9.) RB , AB , to be two axes about which every point in the plane $ABPR$ tends to move with velocities as the respective distances from the axes; let PQ perpendicular to AB be to PR perpendicular to RB as the angular velocity of P about RB to the angular velocity of the same point about AB , and let the velocities be in contrary directions: then, I say, every point in the plane will move with a velocity proportional to its distance from the axis PB . First, it is evident, that any point c in the axis RB will move round PB with a velocity proportional to its distance cm : for the point c lying in the axis RB has no velocity round RB , and cm is proportional to cn . Draw pc parallel to AB , and any point d in that line will move with a velocity proportional to dv , which is perpendicular to PB , for the following reason.

The

The velocity of D is equal to the difference of its two velocities round the respective axes RB , AB , or the difference of D 's velocity round RB , and P 's velocity round AB , since all the points in PC move with the same velocity round AB . Draw DT parallel to CR , and this difference will be proportional to RT , because the velocities of P round RB , AB , are supposed equal to each other, and PT is proportional to DV , and every point in the plane moves round PB with a velocity proportional to its distance; and the same thing may be shewn when any point is taken without the plane $ABCP$.

§ 17. Because any point c in the axis RB moves with the same velocity round PB as it does round AB , the angular velocities round the two axes AB , PB , will be to each other inversely as their respective distances CN , cm ; and because $CN : cm :: PB : PC$ and $PR : PQ :: PC : CB$, it follows, that *the diagonal of the parallelogram $PCBC$ will represent the angular velocity of the revolving plane, when BQ , BC , are taken to each other as the angular velocities of the same plane round those respective axes.*

§ 18. From this it clearly follows, that the reason given by SIMPSON ^(e), in his miscellaneous tracts, of the difference between his own solution and that of NEWTON in the Principia, cannot possibly be the true one. "It

(e) Pages 44. and 45.

" appears:

“ appears further,” says he, “ by perusing his thirty-ninth proposition, that he there assumes it as a principle, that if a ring encompassing the earth at its equator, but detached therefrom, was to tend or begin to move about its diameter with the same accelerative force or angular celerity as that whereby the earth itself tends to move about the same diameter through the action of the Sun, that then the motion of the nodes of the ring and of the equator would be exactly the same.”

The principle is certainly implied in NEWTON's proof, and is capable of the most rigid demonstration, art. 16, 17.

§ 19. It will be asked then, where is the fault of NEWTON's reasoning? How comes his conclusion to be too little by above one half? It is acknowledged on all hands that there is an error in his third lemma; but then the correction of that error makes only a very small alteration in the result.

It is impossible for any one to form a complete judgment of his method without going through the whole of his calculations, which pre-supposes that the mean motion of the lunar nodes is computed. This motion may be concisely determined and exactly enough for the purpose from prop. 30. of the Principia, and from thence is inferred the motion of the nodes of a satellite revolving in

in the plane of the equator at the surface of the earth with a velocity equal to that of the earth round its axis. We are then to suppose, that the *mean* motion of the nodes of a satellite, revolving with such a velocity, is the same with the motion of the nodes of a ring of rigid matter surrounding the earth at its equator, and revolving with the same velocity. This last hypothesis is admitted by SIMPSON, who thinks that his own second lemma contains a full demonstration of the point. For my own part, I believe with FRISIUS, that we are to look here for the material error in NEWTON's solution of the problem. It is evident, that the *true* motion of the nodes of the satellite, and the ring of matter, are not the same; and it is by no means obvious, that their mean motions are so. The mean motion of the nodes of a ring of hard matter cohering together is very easily computed by the method in art. 4th to the 9th, and turns out nearly double the mean motion of a Moon revolving at the surface of the earth with the same velocity.

It is a very interesting enquiry to find out the real cause of the mistake in the Principia, lib. III. prop. 39.; and therefore at a future opportunity I may, perhaps, consider this particular part of the subject more attentively. I have long been satisfied with the account already given, and

should probably have remained so, if M. D'ALEMBERT ^(d), in his Opusc. vol. V. had not persisted to affirm, that the mean motion of the nodes of the ring of matter and of the satellitè were the same.

This opinion of so celebrated a mathematician raises scruples in one's mind; and shews, that when we venture to differ from Sir ISAAC NEWTON in these matters, it is with the utmost difficulty that we can arrive at certainty.

(d) Il n'y a de parité que dans le mouvement *moyen* de ces deux anneaux, ou de l'anneau solide et de la lune; les mouvemens *instantanés* sont très differens de part et d'autre; ainsi la comparaison du mouvement de l'anneau avec celui de la lune, serviroit tout au plus à trouver le mouvement *moyen* de l'anneau, ou de la précession des équinoxes, mais nullement à déterminer la nutation de l'axe et l'équation de la précession.



Fig. 1.

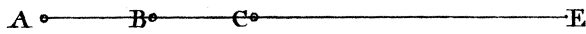


Fig. 2.

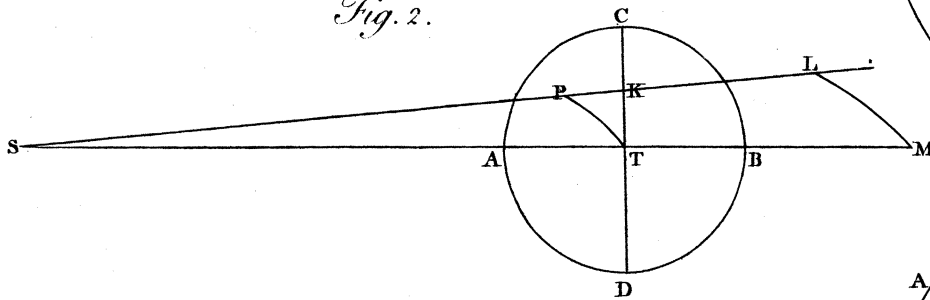


Fig. 3.

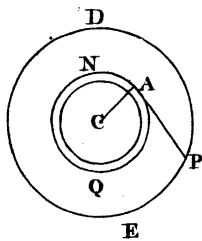


Fig. 4.

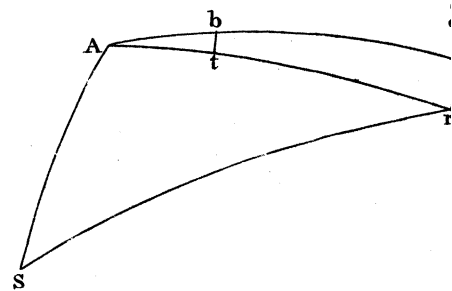
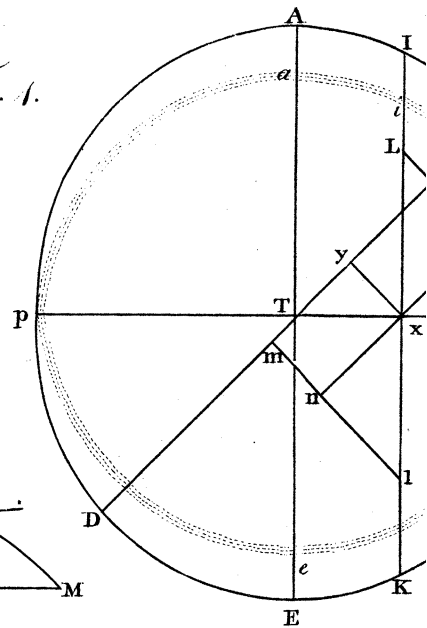
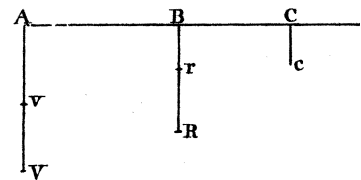


Fig. 6.



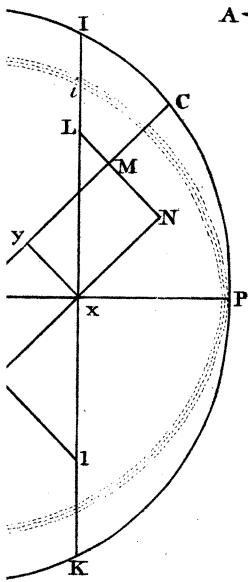
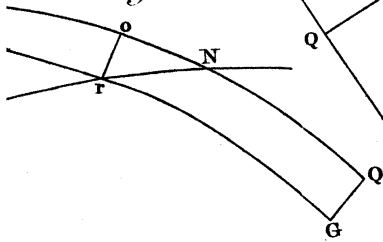


Fig. 5.



6.

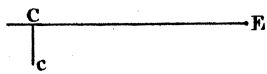


Fig. 7.

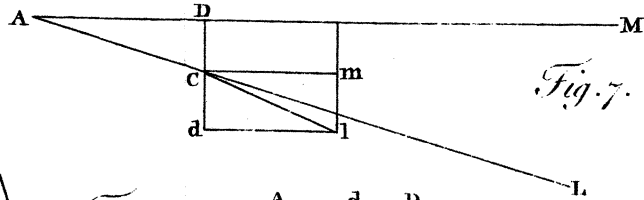


Fig. 8.

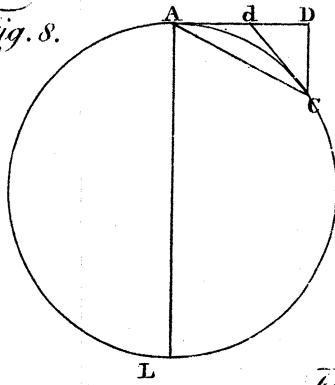


Fig. 9.

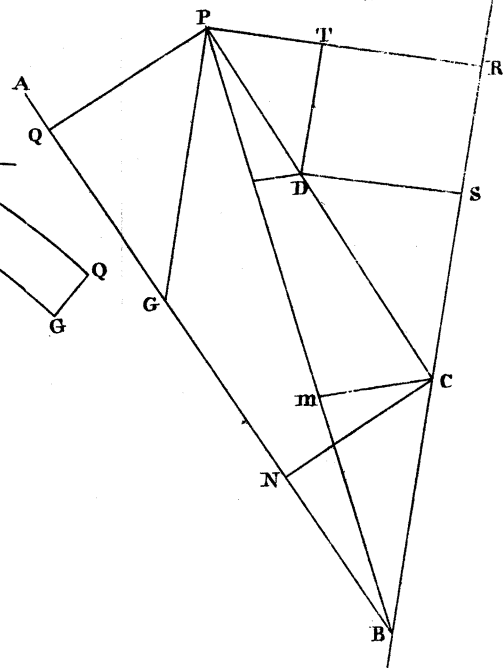


Fig. 1.



Fig. 2.

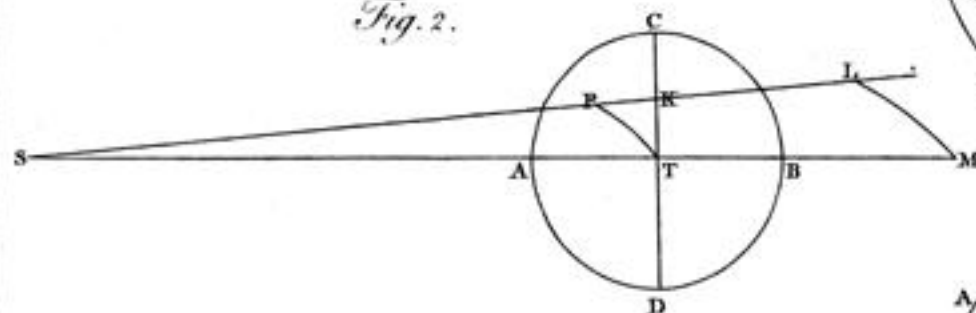


Fig. 3.

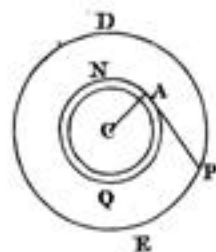


Fig. 4.

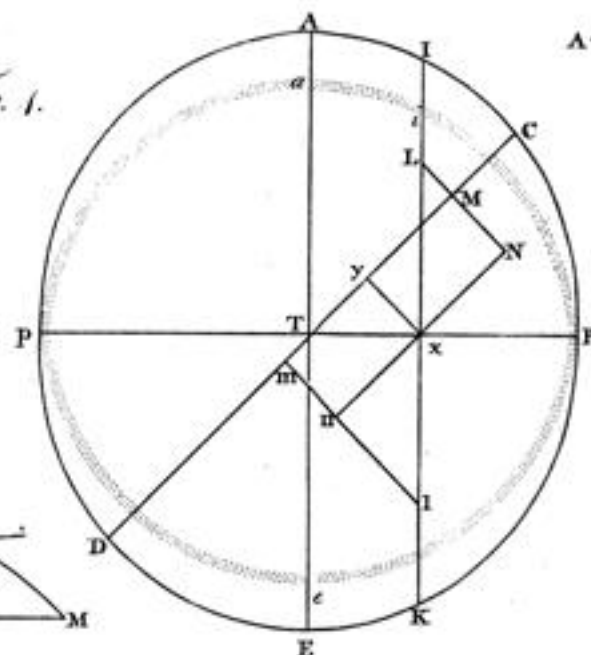


Fig. 5.

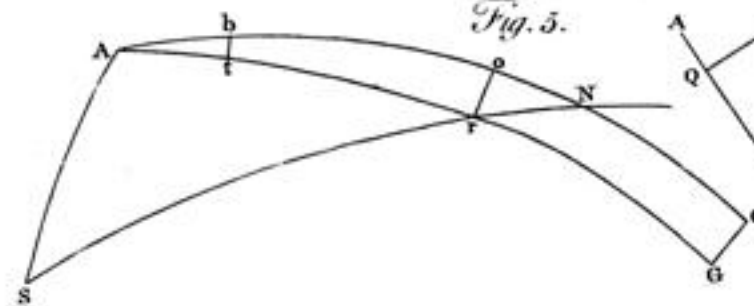


Fig. 6.

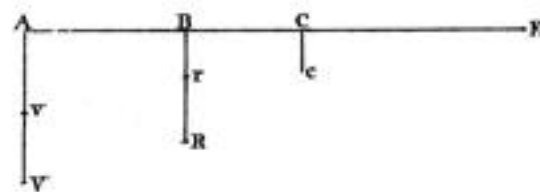


Fig. 7.

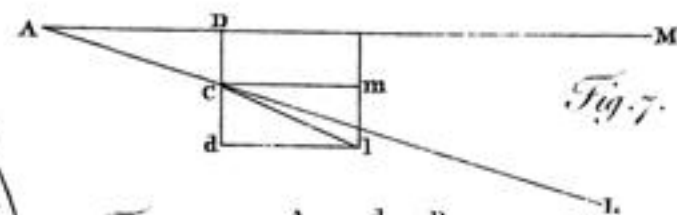


Fig. 8.

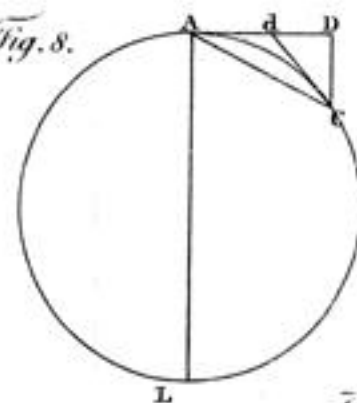


Fig. 9.

